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DYNAMIC NON-LINEAR WAVE PROPAGATION IN IONIZED MEDIA

8. Reflection of electromagnetic waves from a dielectric, plane layer, varying periodically in space and time

BY

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1. Introduction and Summary.

The problem of reflection and refraction of electromagnetic waves by a dielectric medium is an old subject, which first attracted interest in optics. Later on the well known refraction laws, found by Snellius and Fresnel, for a plane boundary, were applied to radio waves, especially short ones. The treatment of these problems can be found in any text book on electromagnetic theory, e.g. Stratton [1].

In the above mentioned cases the dielectric is assumed to be independent of space and time. The interaction between an electromagnetic wave and a medium, the electrical properties of which are varying with space and time has become of growing interest during the last years. The dynamics of non-linear wave propagation in ionized media has been studied in detail by Rydbeck in a number of papers [2], where among other things the complicated case of a magneto-ionic medium is discussed. Wilhelmsson has treated related problems in two reports [3]. Interaction phenomena of similar types has also been investigated by Simon [4] for a dielectric medium, embedded in a wave guide.

The purpose of the investigation, presented in this report, is to study the interaction between an obliquely incident, plane electromagnetic wave and a plane, semi-infinite dielectric medium, the dielectric constant of which contains a space- and time-varying component. The problem of the possible creating such a forced variation, or disturbance, lies outside the scope of this report. In this connection reference is made to Rydbeck [2].

Let us first consider a medium, the dielectric constant of which is periodically fluctuating only in time. It can then be shown that one gets parametric resonance when the angular frequency, ω_1, of the incident electromagnetic wave is equal to half the angular frequency, ω_2, of the forced, or pumped, variations in the disturbed medium. However, when the disturbance is propagating, this resonance condition becomes

altered. We find, that in order to get resonance effects certain relations depending on the angle of incidence and the phase velocities of the incident and "pump"-wave, predicted by Rydbeck [2], must be fulfilled. The harmonic time variation of the dielectric medium produces transmitted and reflected waves ("non-linear" modes) on the characteristic frequencies $\omega_1 \pm n\,\omega_2$, where n = 0, 1, 2 ... The angles of reflection and transmission will be functions of the propagation parameters of the two waves as expressed by a generalisation of Snellius's law. The investigation, finally, also includes the case where losses are taken into account.

I want to express my sincere gratitude to Professor O.E.H.Rydbeck, Head of the Research Laboratory of Electronics, Chalmers University of Technology, Gothenburg for suggesting the problem and for encouraging advice.

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2. The interaction between an obliquely incident, plane electromagnetic wave and a plane semi-infinite medium with the dielectric constant $\varepsilon_2 = \varepsilon_o + \Delta\varepsilon \cos(\omega_2 t - \beta x)$.

Consider a medium in which the dielectric constant ε_2 varies like

$$\varepsilon_2 = \varepsilon_o + \Delta\varepsilon \cos(\omega_2 t - \beta x) \tag{2.1}$$

where ε_o is the permittivity of free space and $\Delta\varepsilon/\varepsilon_o << 1$. This medium (medium 2) will form the upper half-space in a coordinate system explained in Fig. 2.1. The lower half-space is assumed to be vacuum (medium 1). It should be added, in this connection, that it is outside the scope of the present communication, which only deals with the principal interaction aspects, to discuss the physical realizability of medium variations of the nature described by (2.1).

In order to find out in which way a plane TE-wave, with the angular frequency ω_1 and propagation constant $k_o^{(0)}$ obliquely incident from medium 1, [the angle of incidence is $\varphi_1^{(0)}$], will be coupled to the "pumped" medium 2, the wave equation must be solved and the boundary conditions satisfied.

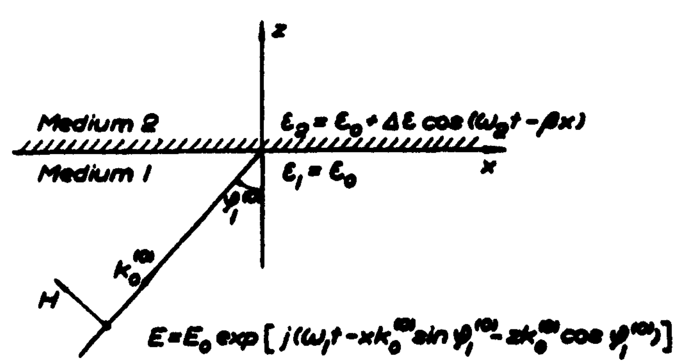


Fig. 2.1. A plane electromagnetic wave obliquely incident upon an oscillating, dielectric medium.

Maxwell's equations in medium 2 can be written

$$\nabla \times \bar{H} = \frac{\partial}{\partial t} \varepsilon_2 \bar{E} \tag{2.2a}$$

$$\nabla \times \bar{E} = -\mu_o \frac{\partial}{\partial t} \bar{H} \tag{2.2b}$$

$$\nabla \varepsilon_2 \bar{E} = 0 \tag{2.2c}$$

$$\nabla \bar{H} = 0. \tag{2.2d}$$

Assuming that $\partial/\partial y = 0$, $E_x = E_z = 0$ and putting $E_y = E$, we get from equations (2.2) and (2.1) the following wave equation

$$\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial z^2} - \frac{1}{c_o^2}\frac{\partial^2 E}{\partial t^2} = \frac{1}{c_o^2}\frac{\Delta\varepsilon}{\varepsilon_o}\frac{\partial^2}{\partial t^2}\left[E \cos(\omega_2 t - \beta x)\right] \tag{2.3}$$

where

$$c_o = \frac{1}{\sqrt{\mu_o \varepsilon_o}} = \frac{\omega_1}{k_o^{(0)}} = \frac{(\omega_1 + n\omega_2)}{k_o^{(n)}}. \tag{2.3a}$$

The solution of (2.3) can be written as a series, namely

$$E = e^{j[\omega_1 t - k_{x_2}^{(0)} x - k_{z_2}^{(0)} z]} \cdot F(\omega_2 t - \beta x), \tag{2.4}$$

where

$$F(\omega_2 t - \beta x) = \sum_{n=-\infty}^{n=+\infty} A^{(n)} e^{jn(\omega_2 t - \beta x)}. \tag{2.5}$$

In equation (2.4) $k_{x_2}^{(0)}$ and $k_{z_2}^{(0)}$ are the x- and z-components of the propagation constant in medium 2 for the wave with angular frequency ω_1. Similarly $k_o^{(0)} \sin\varphi_1^{(0)}$, and $k_o^{(0)} \cos \varphi_1^{(0)}$

are denoted $k_{x_1}^{(0)}$ and $k_{z_1}^{(0)}$, respectively, etc. Since we have assumed $\Delta\varepsilon/\varepsilon_o \ll 1$, we neglect second and higher order terms in $\Delta\varepsilon/\varepsilon_o$. The first order solution will then consist of three components with the amplitudes $A^{(0)}$, $A^{(+1)}$ and $A^{(-1)}$. In the first order $A^{(0)}$ is determined by the incident wave only. $A^{(+1)}$ and $A^{(-1)}$ are then found from equation (2.3) to be

$$A^{(+1)} = \frac{\Delta\varepsilon}{2\varepsilon_o} \cdot \frac{(\omega_1 + \omega_2)^2}{c_o^2\beta[\beta + 2k_{x_2}^{(0)}] - \omega_2(\omega_2 + 2\omega_1)} \cdot A^{(0)}, \quad (2.6)$$

$$A^{(-1)} = \frac{\Delta\varepsilon}{2\varepsilon_o} \cdot \frac{(\omega_1 - \omega_2)^2}{c_o^2\beta[\beta - 2k_{x_2}^{(0)}] - \omega_2(\omega_2 - 2\omega_1)} \cdot A^{(0)}. \quad (2.7)$$

When the disturbance is not propagating, i.e. when $\beta = 0$, parametric resonance will occur for $\omega_2 = 2\omega_1$. In our case, with $\beta \neq 0$, the condition $\beta = 2k_{x_2}^{(0)}$ must also be fulfilled in order to get a resonance of $A^{(-1)}$ in equation (2.7) at $\omega_2 = 2\omega_1$. We will discuss related questions in some more detail in the next paragraph.

The boundary conditions are (index 1 refers to medium 1, etc)

$$\left.\begin{aligned} E_1 &= E_2 \\ \frac{\partial E_1}{\partial z} &= \frac{\partial E_2}{\partial z} \\ \frac{\partial E_1}{\partial x} &= \frac{\partial E_2}{\partial x} \end{aligned}\right\} \qquad (z = 0) \qquad (2.8)$$

In order to satisfy equations (2.8) we write the total fields in the two media, considering contributions <u>of all orders</u> in $\Delta\varepsilon/\varepsilon_o$, in the following way, viz.

$$
\left.
\begin{aligned}
E_1 = E_{inc} \exp\Big\{ j\big[\omega_1 t - x\underbrace{k_o^{(0)}\sin\varphi_1^{(0)}}_{k_{x_1}^{(0)}} - z\underbrace{k_o^{(0)}\cos\varphi_1^{(0)}}_{k_{z_1}^{(0)}}\big]\Big\} + \\
+ \sum_{n=-\infty}^{n=+\infty} R^{(n)} \exp\Big(j\Big\{(\omega_1 + n\omega_2)t - \\
- x[k_o^{(0)}\sin\varphi_1^{(0)} + n\beta] + z\underbrace{k_o^{(n)}\cos\varphi_1^{(n)}}_{k_{z_1}^{(n)}}\Big\}\Big)
\end{aligned}
\right\} (2.9)
$$

and

$$E_2 = F(\omega_2 t - \beta x) \sum_{m=-\infty}^{m=+\infty} b^{(m)} \exp\Big\{ j\big[(\omega_1 + m\omega_2)t - xk_{x_2}^{(m)} - z\,k_{z_2}^{(m)}\big]\Big\} . \tag{2.10}$$

In equation (2.9) $\varphi_1^{(n)}$ is the angle of reflection for the wave with angular frequency $\omega_1 + n\,\omega_2$, $k^{(n)}$ [with components $k_{x_2}^{(n)}$ and $k_{z_2}^{(n)}$] denotes the k-value of the $\omega_1 + n\,\omega_2$ wave in medium 2, and $k_1^{(n)} = k_o^{(n)}$ [with components $k_{x_1}^{(n)}$ and $k_{z_1}^{(n)}$] the corresponding quantity in medium 1.

Inserting the expressions (2.9) and (2.10) into (2.8) yields

$$A^{(0)}\,b^{(0)} = E_{inc} \frac{2k_o^{(0)}\cos\varphi_1^{(0)}}{k_o^{(0)}\cos\varphi_1^{(0)} + k_{z_2}^{(0)}} = E_{inc} \frac{2k_{z_1}^{(0)}}{k_{z_1}^{(0)} + k_{z_2}^{(0)}}, \tag{2.11a}$$

$$R^{(0)} = E_{inc} \frac{k_o^{(0)}\cos\varphi_1^{(0)} - k_{z_2}^{(0)}}{k_o^{(0)}\cos\varphi_1^{(0)} + k_{z_2}^{(0)}} = E_{inc} \frac{k_{z_1}^{(0)} - k_{z_2}^{(0)}}{k_{z_1}^{(0)} + k_{z_2}^{(0)}}. \tag{2.11b}$$

These expressions we recognize as the well-known Fresnel formulae. The first order coefficients are found to be

$$A^{(0)}b^{(+1)} = E_{inc}\,\frac{\Delta\varepsilon}{2\varepsilon_o}\cdot\frac{(\omega\ \ +\omega_2)^2}{\omega_2(\omega_2+2\omega_1)-c_o^2\beta[\beta+2k_{x_2}^{(0)}]}\ x$$

$$x\,\frac{k_{z_1}^{(+1)}+k_{z_2}^{(0)}}{k_{z_1}^{(+1)}+k_{z_2}^{(+1)}}\cdot\frac{2k_{z_1}^{(0)}}{k_{z_1}^{(0)}+k_{z_2}^{(0)}}, \qquad (2.12a)$$

$$R^{(+1)} = E_{inc}\,\frac{\Delta\varepsilon}{2\varepsilon_o}\cdot\frac{(\omega_1+\omega_2)^2}{\omega_2(\omega_2+2\omega_1)-c_o^2\beta[\beta+2k_{x_2}^{(0)}]}\ x$$

$$x\,\frac{k_{z_2}^{(0)}-k_{z_2}^{(+1)}}{k_{z_1}^{(+1)}+k_{z_2}^{(+1)}}\cdot\frac{2k_{z_1}^{(0)}}{k_{z_1}^{(0)}+k_{z_2}^{(0)}}, \qquad (2.12b)$$

$$A^{(0)}b^{(-1)} = E_{inc}\,\frac{\Delta\varepsilon}{2\varepsilon_o}\cdot\frac{(\omega_1-\omega_2)^2}{\omega_2(\omega_2-2\omega_1)-c_o^2\beta[\beta-2k_{x_2}^{(0)}]}\ x$$

$$x\,\frac{k_{z_1}^{(-1)}+k_{z_2}^{(0)}}{k_{z_1}^{(-1)}+k_{z_2}^{(-1)}}\cdot\frac{2k_{z_1}^{(0)}}{k_{z_1}^{(0)}+k_{z_2}^{(0)}}, \qquad (2.13a)$$

$$R^{(-1)} = E_{inc}\,\frac{\Delta\varepsilon}{2\varepsilon_o}\cdot\frac{(\omega_1-\omega_2)^2}{\omega_2(\omega_2-2\omega_1)-c_o^2\beta[\beta-2k_{x_2}^{(0)}]}\ x$$

$$x\,\frac{k_{z_2}^{(0)}-k_{z_2}^{(-1)}}{k_{z_1}^{(-1)}+k_{z_2}^{(-1)}}\cdot\frac{2k_{z_1}^{(0)}}{k_{z_1}^{(0)}+k_{z_2}^{(0)}}. \qquad (2.13b)$$

The sign of the $k_{z_2}^{(-1)}$-terms must sometimes be changed. This is discussed on page 9.

When determining the b- and R-values we have considered only the amplitudes of the different waves. The relevant transverse phase terms, however, must also be equal for the various n-values in order to completely fulfil the boundary conditions. This yields the following relations between the k-values

$$\left.\begin{aligned} k_{x_2}^{(n)} &= k_o^{(0)} \sin \varphi_1^{(0)} + n\beta \qquad &(2.14a) \\ k_{z_2}^{(n)} &= k_o^{(n)} \cos \varphi_2^{(n)} \qquad &(2.14b) \end{aligned}\right\} \quad (n = 0, \pm 1, \pm 2, ..)$$

where $\varphi_2^{(n)}$ is the angle of refraction for the $\omega_1 + n\omega_2$ wave. (Note that in this case $\varphi_1^{(n)} = \varphi_2^{(n)}$ because the mean value of ε_2 is equal to ε_o.)

It is important to note that in the first order the coupling between the incident wave and the disturbed, or pumped, medium gives rise to two different waves of the same angular frequency, $\omega_1 + \omega_2$ or $\omega_1 - \omega_2$, in the disturbed medium. If we restrict the discussion to the difference frequency, $\omega_1 - \omega_2$, wave, we find that both waves must have the same k-component in the x-direction because of the boundary conditions at $z = 0$. In the z-direction one wave will have the k-value $k_{z2} = k_{z2}^{(-1)}$ and the other one $k_{z2} = k_{z2}^{(0)}$. The amplitudes of the two waves are $A^{(0)}\ b^{(-1)}$ for the former and $A^{(-1)}\ b^{(0)}$ for the latter. Thus both wave components travel with the same phase velocity in the x-direction, which is a characteristic feature. The angles of refraction, as well as the phase velocities in the direction of the wave normals, are different. These angles are determined by the following relations

$$\left(\sin \varphi_2^{(-1)}\right)_{k_{z_2}=k_{z_2}^{(-1)}} = \frac{\sin \varphi_1^{(0)} - \frac{\beta}{k_o^{(0)}}}{1 - \frac{\omega_2}{\omega_1}} = \frac{k_{x_1}^{(0)} - \beta}{k_o^{(-1)}}, \qquad (2.15a)$$

when $k_{z_2} = k_{z_2}^{(-1)}$, and

$$\left(\sin \varphi_2^{(-1)}\right)_{k_{z_2}=k_{z_2}^{(0)}} = \frac{\sin \varphi_1^{(0)} - \frac{\beta}{k_o^{(0)}}}{\left\{1-\beta \frac{[2k_o^{(0)} \sin \varphi_1^{(0)} - \beta]}{[k_o^{(0)}]^2}\right\}^{\frac{1}{2}}} =$$

$$= \frac{k_{x_1}^{(0)} - \beta}{\left\{[k_o^{(0)}]^2 - \beta[2k_{x_1}^{(0)} - \beta]\right\}^{\frac{1}{2}}} = \frac{k_{x_1}^{(0)} - \beta}{\left\{[k_{x_1}^{(0)} - \beta]^2 + [k_{z_1}^{(0)}]^2\right\}^{\frac{1}{2}}} \tag{2.15b}$$

when $k_{z_2} = k_{z_2}^{(0)}$.

The k_{z_2}-values are

$$[k_{z_2}^{(-1)}]^2 = \frac{(\omega_1 - \omega_2)^2}{c_o^2} - [k_{x_2}^{(0)} - \beta]^2, \tag{2.16a}$$

and

$$[k_{z_2}^{(0)}]^2 = \frac{\omega_1^2}{c_o^2} - [k_{x_2}^{(0)}]^2. \tag{2.16b}$$

In this connection it must be pointed out that the sign of $k_{z_2}^{(-1)}$ must be changed when $\omega_2 > \omega_1$ in order to describe the $\omega_2 - \omega_1$ waves properly. This change must be made in equations (2.13) and in all the following formulas of the same kind.

The value of $[\sin \varphi_2^{(-1)}]_{k_{z_2}=k_{z_2}^{(-1)}}$ will become > 1, if $\sin \varphi_1^{(0)} > \beta/k_o^{(0)} + 1 - \omega_2/\omega_1$, or < -1, if $\sin \varphi_1^{(0)} < \beta/k_o^{(0)} - (1 - \omega_2/\omega_1)$. Equation (2.15a) can in these cases be satisfied only by complex values of $[\varphi_2^{(-1)}]_{k_{z_2}=k_{z_2}^{(-1)}}$. When this

is the case, only surface waves, exponentially decreasing in the z-direction, are produced in the pumped medium at the difference frequency. In the disturbed medium the field will decay like

$$\exp\left\{-\, z\, k_o^{(-1)}\, \left[\left(\frac{k_{x_1}^{(0)} - \beta}{k_o^{(-1)}}\right)^2 - 1\right]^{\frac{1}{2}}\right\}. \qquad (2.17)$$

It should be noted in this connection, that the absolute value of

$$\left(\sin \varphi_2^{(-1)}\right)_{k_{z_2}=k_{z_2}^{(0)}}$$

can never become > 1 for $|\varphi_1^{(0)}| < 90^\circ$.

In the reflected field there is only one component for each frequency. The amplitudes are given by the corresponding R-values. The angle of reflection for the $\omega_1 - \omega_2$ wave is

$$\sin \varphi_1^{(-1)} = \frac{\sin \varphi_1^{(0)} - \frac{\beta}{k_o^{(0)}}}{1 - \frac{\omega_2}{\omega_1}} = \frac{\frac{[k_{x_1}^{(0)} - \beta]}{k_o^{(0)}}}{1 - \frac{\omega_2}{\omega_1}} = \frac{k_{x_1}^{(0)} - \beta}{k_o^{(-1)}}. \qquad (2.18)$$

Let us assume that $\beta = 0$. It is interesting and important to note that we, in the (degenerate) parametric resonance case $\omega_2 = 2\omega_1$, then get a reflected wave travelling in <u>opposite</u> direction to that of the incident wave. When $\beta \neq 0$ we again obtain a wave in the opposite direction, if $\omega_2/\omega_1 = 2 - \beta/k_{x_1}^{(0)}$. The amplitude of this wave is finite whereas in the resonance case the amplitude is infinite in the first order if we neglect losses [see equation (2.7)].

Taking into account higher order terms in $\Delta\varepsilon/\varepsilon_o$ we get $n + 1$ ($n = 1, 2, 3, \ldots$) wave components in the disturbed medium for the field with angular frequencies $\omega_1 \pm n\,\omega_2$. Each one of these components will have a phase

velocity and angle of refraction, determined by the k-components $k_{x_2}^{(0)} \pm n\beta$ in the x-direction and $k_{z_2}^{(\pm l)}$ ($l = 0, 1, 2, \ldots n$) in the z-direction. The k_{z2}-values are defined by

$$k_{z_2}^{(\pm l)} = \left\{ \frac{(\omega_1 \pm l\, \omega_2)^2}{c_o^2} - [k_{x_2}^{(0)} \pm l\beta]^2 \right\}^{\frac{1}{2}}. \qquad (2.19)$$

The reflected field contains only one component for each frequency because no coupling takes place in medium 1. The angles of reflection become

$$\sin \varphi_1^{(\pm n)} = \frac{\sin \varphi_1^{(0)} \pm \frac{n\beta}{k_o^{(0)}}}{1 \pm \frac{n\,\omega_2}{\omega_1}} = \frac{\frac{k_{x_1}^{(0)} \pm n\beta}{k_o^{(0)}}}{1 \pm \frac{n\omega_2}{\omega_1}} \quad (n \geq 0) \quad (2.20)$$

The transmission and reflection conditions are discussed in more detail in the next paragraph.

3. **The interaction between an obliquely incident, plane electromagnetic wave and a plane semi-infinite medium with the dielectric constant $\varepsilon_2 = \varepsilon + \Delta\varepsilon \cos(\omega_2 t - \beta x)$.**

Now consider a medium with the same properties as before but with the mean value of the dielectric constant $\varepsilon_2 \neq \varepsilon_o$. Using the same technique as in chapter 2 and with the same assumptions, we obtain the following wave equation

$$\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial z^2} - \frac{1}{c_2^2}\frac{\partial^2 E}{\partial t^2} = \frac{1}{c_2^2}\frac{\Delta\varepsilon}{\varepsilon}\frac{\partial^2}{\partial t^2}\left[E \cos(\omega_2 t - \beta x)\right], \tag{3.1}$$

where

$$c_2 = \frac{1}{\sqrt{\mu_o \varepsilon}}; \quad \varepsilon_2 = \varepsilon + \Delta\varepsilon \cos(\omega_2 t - \beta x). \quad \varepsilon \neq \varepsilon_o.$$

The solution of equation (3.1) will be a series of the same form as (2.4) and the A-values are now found to be

$$A^{(+1)} = \frac{\Delta\varepsilon}{2\varepsilon}\,\frac{(\omega_1+\omega_2)^2}{c_2^2\beta[\beta+2k_{x_2}^{(0)}] - \omega_2(\omega_2+2\omega_1)}\,A^{(0)}, \tag{3.2a}$$

$$A^{(-1)} = \frac{\Delta\varepsilon}{2\varepsilon}\,\frac{(\omega_1-\omega_2)^2}{c_2^2\beta[\beta-2k_{x_2}^{(0)}] - \omega_2(\omega_2-2\omega_1)}\,A^{(0)}. \tag{3.2b}$$

The b- and R-values will formally be the same as in the case $\varepsilon = \varepsilon_o$, the only difference being that the factor $\Delta\varepsilon/\varepsilon$ replaces $\Delta\varepsilon/\varepsilon_o$ and c_2 replaces c_o.

The k-values in medium 2 are

$$\left.\begin{aligned} k_{x_2}^{(n)} &= k_o^{(0)} \sin \varphi_1^{(0)} + n\beta && (3.3a)\\ k_{z_2}^{(n)} &= k_2^{(n)} \cos \varphi_2^{(n)} && (3.3b)\end{aligned}\right\} (n = 0, \pm 1, \pm 2, \ldots)$$

where

$$k_2^{(n)} = \frac{\omega_1 + n\omega_2}{c_2} = k_o^{(n)}\sqrt{\frac{\varepsilon}{\varepsilon_o}}.$$

The angles of reflection for the different waves are independent of ε and the expression for them is given in equation (2.20). The angles of refraction, however, will change depending upon the different phase velocities. We obtain for the ω_1 wave

$$\sin\varphi_2^{(0)} = \sqrt{\frac{\varepsilon_o}{\varepsilon}}\sin\varphi_1^{(0)} \tag{3.4}$$

and for the $\omega_1 - \omega_2$ waves

$$\left(\sin\varphi_2^{(-1)}\right)_{k_{z_2}=k_{z_2}^{(-1)}} = \sqrt{\frac{\varepsilon_o}{\varepsilon}}\,\frac{\sin\varphi_1^{(0)} - \dfrac{\beta}{k_o^{(0)}}}{1-\dfrac{\omega_2}{\omega_1}} =$$

$$= \sqrt{\frac{\varepsilon_o}{\varepsilon}}\,\frac{\dfrac{(k_{x_1}^{(0)}-\beta)}{k_o^{(0)}}}{1-\dfrac{\omega_2}{\omega_1}}, \tag{3.5a}$$

$$\left(\sin\varphi_2^{(-1)}\right)_{k_{z_2}=k_{z_2}^{(0)}} = \frac{\sin\varphi_1^{(0)} - \dfrac{\beta}{k_o^{(0)}}}{\left\{\dfrac{\varepsilon}{\varepsilon_o} - \dfrac{\beta[2k_o^{(0)}\sin\varphi_1^{(0)}-\beta]}{[k_o^{(0)}]^2}\right\}^{\frac{1}{2}}} =$$

$$= \frac{k_{x_1}^{(0)}-\beta}{\left\{[k_{x_1}^{(0)}-\beta]^2 + [k_{z_2}^{(0)}]^2\right\}^{\frac{1}{2}}} \tag{3.5b}$$

where we have used the same notations as in chapter 2.

In Fig. 3.1 is sketched the direction of the wave normals of the reflected $\omega_1 - \omega_2$ wave for different values of ω_2/ω_1. We have assumed $\varphi_1^{(0)} = 30^\circ$, $\beta = \omega_2/c_2$ and

$\varepsilon/\varepsilon_o = 1.44$. (Note that in this case $\varphi_1^{(-1)}$ depends on ε_2, because $\beta = \omega_2/c_2$.)

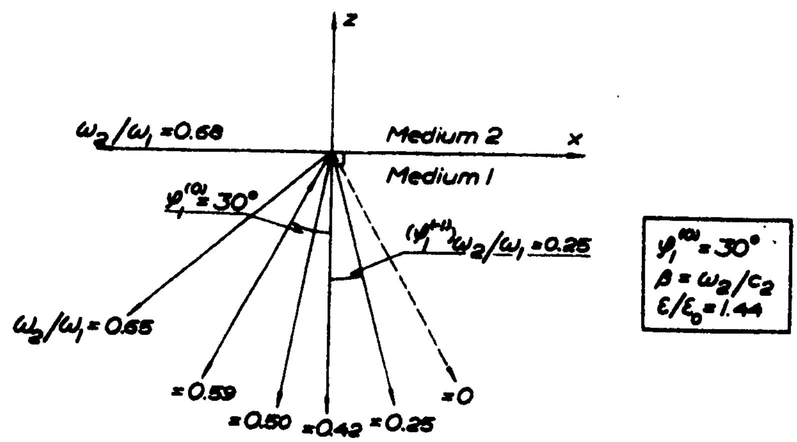


Fig. 3.1. The direction of the wave normal, $\varphi_1^{(-1)}$, for the reflected wave of angular frequency $\omega_1 - \omega_2$, assuming different values of ω_2/ω_1, when $\varepsilon/\varepsilon_o = 1.44$ and $\varphi_1^{(0)} = 30°$.

A reflected wave of angular frequency $\omega_1 - \omega_2$ can exist as a travelling wave only if $\omega_2/\omega_1 < [\sin \varphi_1^{(0)} + 1]/(1 + \sqrt{\varepsilon/\varepsilon_o})$ (when $\beta = \omega_2/c_2$). In other cases we get evanescent, or surface waves. The $\omega_1 + \omega_2$ wave, however, exists as a reflected, travelling wave for all values of $\omega_2/\omega_1 < [1 - \sin \varphi_1^{(0)}]/(\sqrt{\varepsilon/\varepsilon_o} - 1)$, and the angles of reflexion lie between $\varphi_1^{(0)}$ and $+90°$.

The transmitted components of the $\omega_1 - \omega_2$ wave show somewhat different properties. The one with $k_{z2} = k_{z2}^{(-1)}$ behaves similarly to the reflected wave and exists only for

$$\frac{\omega_2}{\omega_1} < \frac{1 + \sqrt{\frac{\varepsilon_o}{\varepsilon}} \sin \varphi_1^{(0)}}{2\sqrt{\frac{\varepsilon_o}{\varepsilon}}},$$

provided that $\beta = \omega_2/c_2$. Fig. 3.2 shows the direction of the wave normals for these waves.

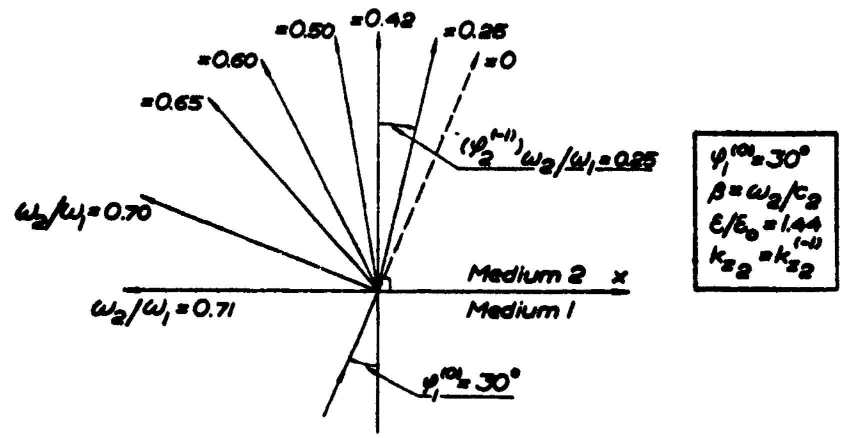


Fig. 3.2. The direction of the wave normal, $\varphi_2^{(-1)}$, for the transmitted wave of angular frequency $\omega_1 - \omega_2$ and $k_{z2} = k_{z2}^{(-1)}$, assuming different values of ω_2/ω_1, when $\varepsilon/\varepsilon_o = 1.44$, and $\varphi_1^{(0)} = 30^\circ$.

For the $\omega_1 - \omega_2$ wave with $k_{z_2} = k_{z_2}^{(0)}$ we get transmitted waves for <u>every</u> value of ω_2/ω_1, and we have in Fig. 3.3 shown the direction of the wave normal in this case.

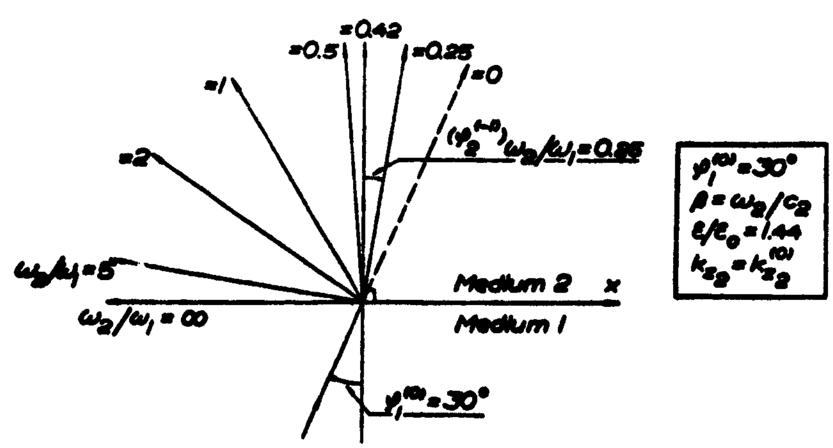


Fig. 3.3. The direction of the wave normal, $\varphi_2^{(-1)}$, for the transmitted wave of angular frequency $\omega_1 - \omega_2$ and $k_{z2} = k_{z2}^{(0)}$, assuming different values of ω_2/ω_1, when $\varepsilon/\varepsilon_o = 1.44$, and $\varphi_1^{(0)} = 30^\circ$.

As far as the amplitudes of the different wave components are concerned we restrict ourselves to the most interesting ones, namely those with angular frequency $\omega_1 - \omega_2$. For the simplest case, namely when the mean value of ε_2 is equal to ε_0 and the forced disturbance in medium 2 does not propagate ($\beta = 0$), the first order amplitude $R^{(-1)}$ of the reflected $\omega_1 - \omega_2$ wave is shown in Fig. 3.4 as a function of ω_2/ω_1.

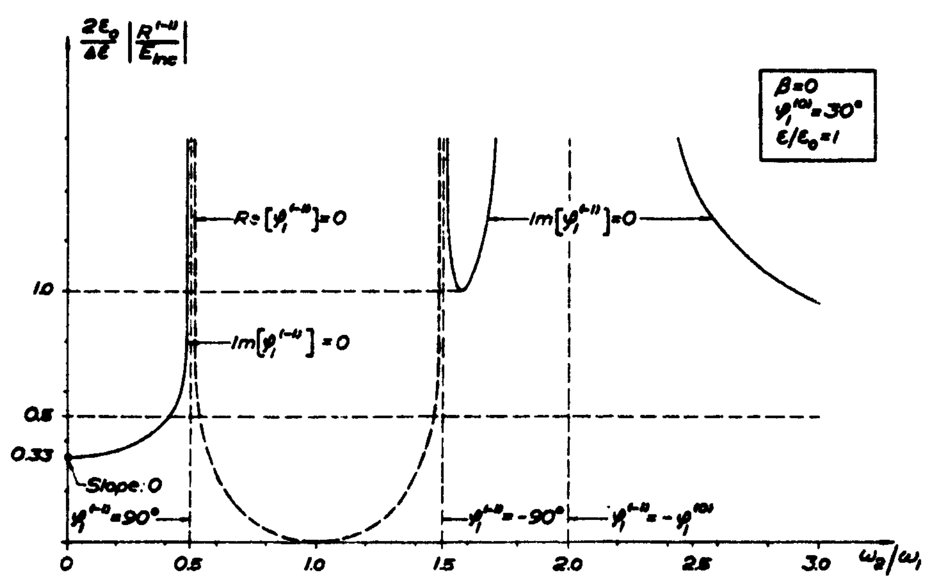


Fig. 3.4. The first order amplitude $R^{(-1)}$ of the reflected $\omega_1 - \omega_2$ wave as a function of ω_2/ω_1, when $\beta = 0$, and $\varphi_1^{(0)} = 30°$.

In the region $0.5 < \omega_2/\omega_1 < 1.5$ $|\sin \varphi_1^{(-1)}| > 1$, i.e. only a surface wave exists in the same, and the amplitude of this wave is shown by the dotted curve in Fig. 3.4. The figure shows that we get regular parametric resonance (which is finite in the second order theory) when $\omega_2/\omega_1 = 2$, as expected. We also get two other resonances, namely when $\omega_2/\omega_1 = 0.5$ and 1.5. These resonances, which take place when $|\varphi_1^{(-1)}| = 90°$, i.e. when $k_{z_1}^{(-1)} = k_0^{(-1)} \cos \varphi_1^{(-1)} =$

$= 0 = k_{z2}^{(-1)}$, or in the general case of $\beta \neq 0$, when $\pm k_{x1}^{(-1)} =$ $= k_{x1}^{(0)} - \beta = k_o^{(0)}(1 - \frac{\omega_2}{\omega_1})$, if $\varepsilon = \varepsilon_o$ ("travelling wave resonance", Rydbeck [2]), are also of parametric nature. In this case, $\varphi_1^{(0)} = 30^\circ$ and $\beta = 0$, they occur when $\omega_1 - \omega_2 =$ $= \pm \omega_1/2$, whereas the "ordinary" parametric resonance occurs when $\omega_1 - \omega_2 = - \omega_2/2$.

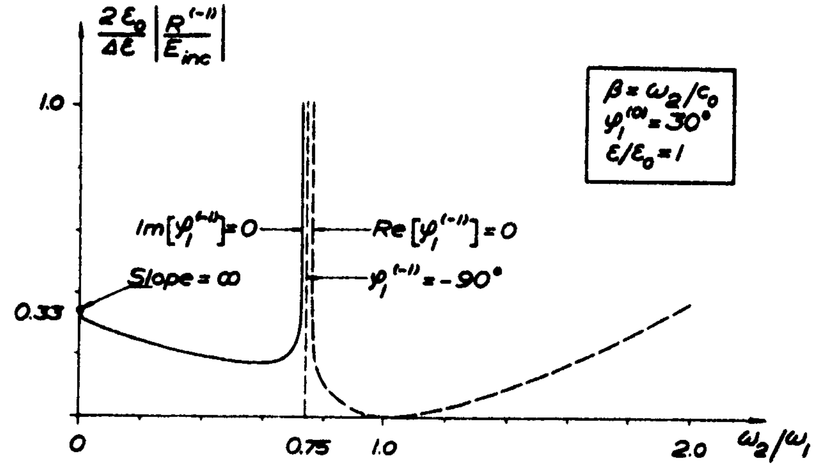


Fig. 3.5. The first order amplitude $R^{(-1)}$ of the reflected $\omega_1 - \omega_2$ wave as a function of ω_2/ω_1, when $\beta = \omega_2/c_o$, and $\varphi_1^{(0)} = 30^\circ$.

In the next case we still assume that the mean value of ε_2 is ε_o but assume that $\beta = \omega_2/c_o$. The first order amplitude $R^{(-1)}$ of the reflected $\omega_1 - \omega_2$ wave now varies with ω_2/ω_1 as shown in Fig. 3.5. The "drastic" influence of a drifting disturbance ($\beta \neq 0$) is evident; the previous "travelling wave" resonance point $\omega_2/\omega_1 = 0.5$ now is changed to $\omega_2/\omega_1 = 0.75$, according to the relation $\omega_1 - \omega_2 = -(\omega_1/2 - \omega_2)$.

Finally, in order to demonstrate the influence of the $\varepsilon/\varepsilon_o$-ratio, we have in Fig. 3.6 plotted the first order amplitude $R^{(-1)}$ of the reflected $\omega_1 - \omega_2$ wave as a

function of ω_2/ω_1, when $\varepsilon/\varepsilon_o = 1.44$ and $\beta = \omega_2/c_2$. It is interesting to note that in this case the amplitude <u>always</u> remains finite.

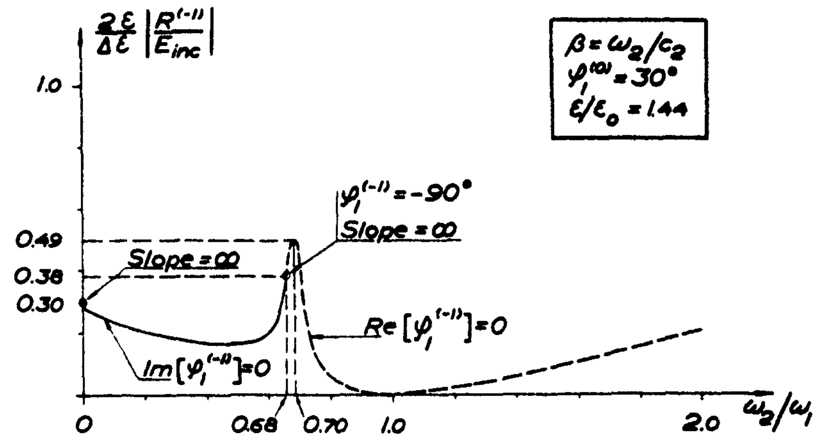


Fig. 3.6. The first order amplitude $R^{(-1)}$ of the reflected $\omega_1 - \omega_2$ wave as a function of ω_2/ω_1, when $\beta = \omega_2/c_2$, $\varepsilon/\varepsilon_o = 1.44$, and $\varphi_1^{(0)} = 30°$.

In Figs. 3.7-3.9 are sketched the amplitudes of the different refracted $\omega_1 - \omega_2$ waves for the same values of β and ε as for the reflected waves.

From Fig. 3.4 we noted that the reflected wave experienced parametric resonance when $\omega_2 = 2\omega_1$, if $\beta = 0$. As far as the refracted waves are concerned this resonance effects occurs only for the wave with $k_{z2} = k_{z2}^{(0)}$ as shown in Fig. 3.7. The other two resonance points, $\omega_2/\omega_1 = 0.5$ and 1.5, will occur only for the wave with $k_{z2} = k_{z2}^{(-1)}$ as also shown in Fig. 3.7.

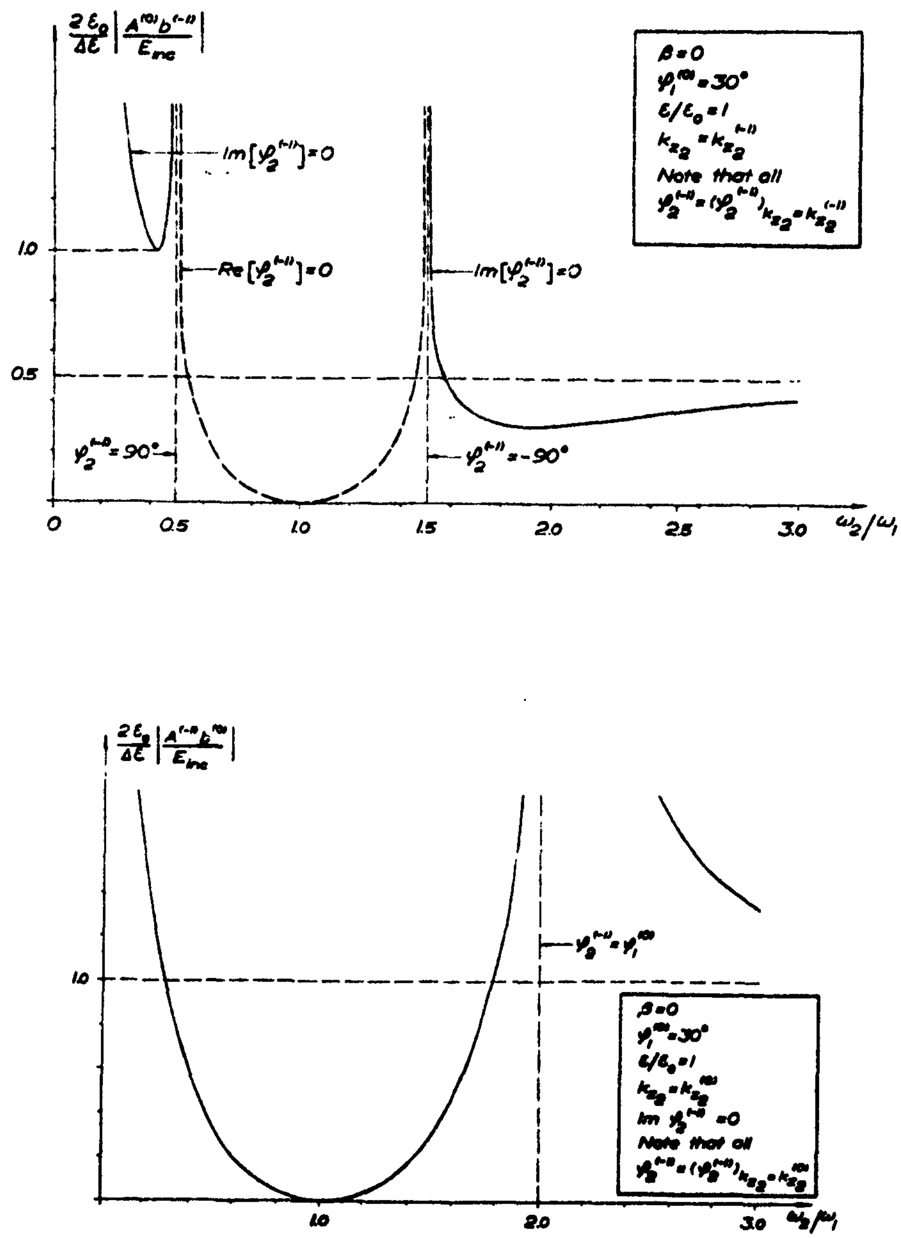
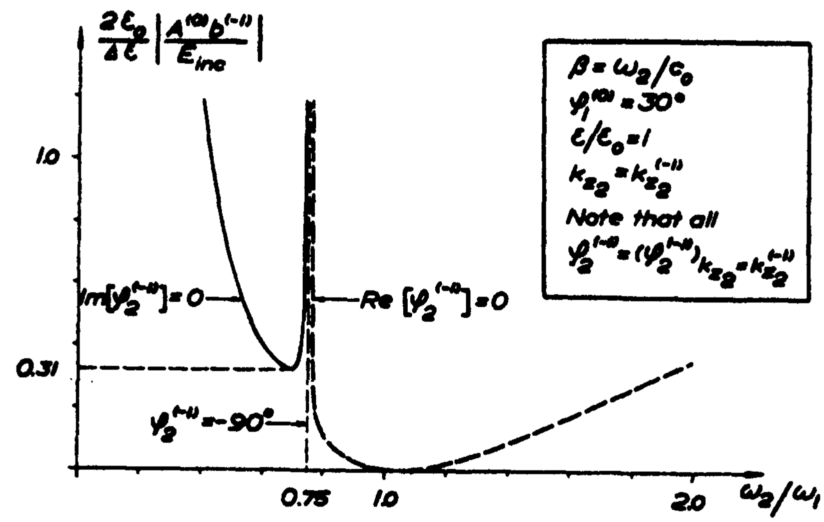


Fig. 3.7. The first order amplitudes $A^{(0)}$ $b^{(-1)}$ and $A^{(-1)}$ $b^{(0)}$ of the transmitted $\omega_1 - \omega_2$ waves as functions of ω_2/ω_1, when $\beta = 0$, and $\varphi_1^{(0)} = 30°$.



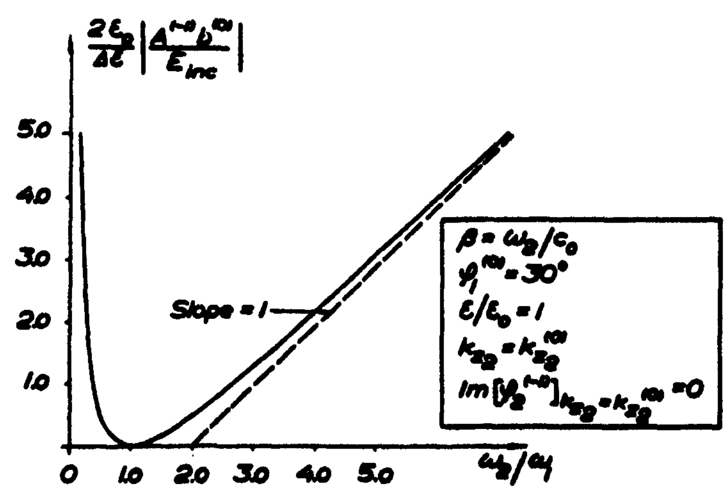
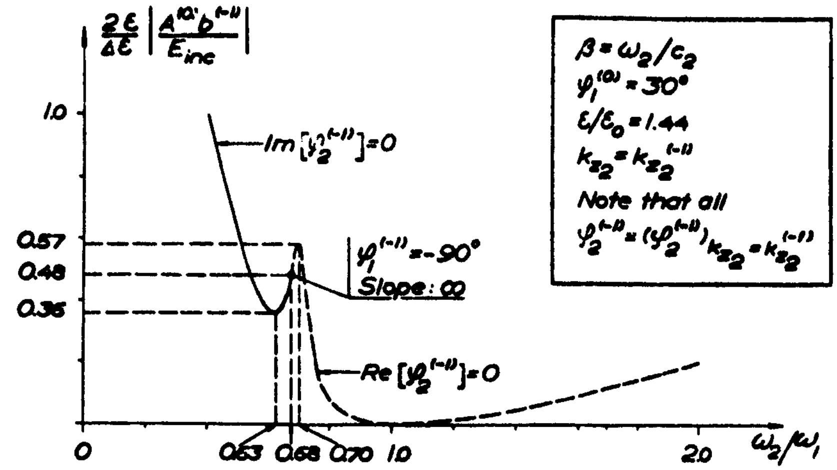


Fig. 3.8. The first order amplitudes $A^{(0)}\,b^{(-1)}$ and $A^{(-1)}\,b^{(0)}$ of the transmitted $\omega_1 - \omega_2$ waves as functions of ω_2/ω_1, when $\beta = \omega_2/c_e$, and $\varphi_1^{(0)} = 30°$.



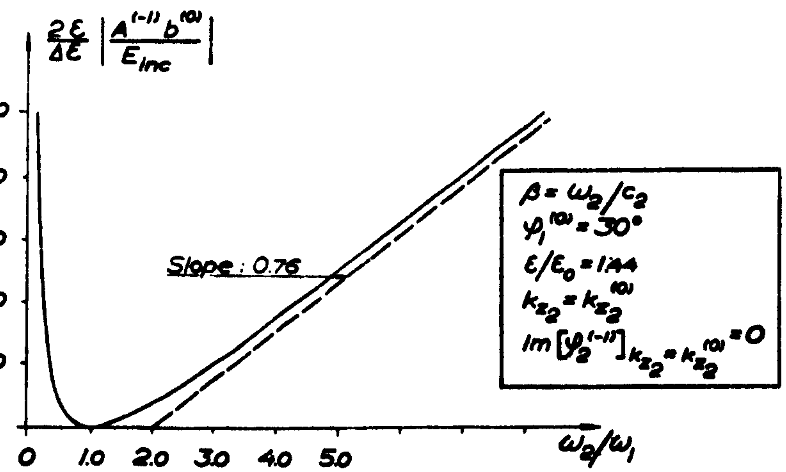


Fig. 3.9. The first order amplitudes $A^{(0)}\ b^{(-1)}$ and $A^{(-1)}\ b^{(0)}$ of the transmitted $\omega_1 - \omega_2$ waves as functions of ω_2/ω_1, when $\beta = \omega_2/c_2$, $\varepsilon/\varepsilon_o = 1.44$, and $\varphi_1^{(0)} = 30°$.

4. The interaction between an obliquely incident, plane electromagnetic wave and a plane semi-infinite medium with the dielectric constant $\varepsilon_2 = \varepsilon - j\sigma/\omega + \Delta\varepsilon\cos(\omega_2 t - \beta x)$.

We have so far only considered media without losses. In many practical cases, however, we must also take into account the conductivity of the medium. If we denote the conductivity by σ and the dielectric constant by ε the complex dielectric constant ε' of the medium is defined by the formula

$$\varepsilon' = \varepsilon - j\sigma/\omega .$$

Our calculations can now be done in the same way as in the lossless case, only replacing ε by ε'.

Using the same notations as before the complex dielectric constant of medium 2 (see Fig. 4.1) can be written

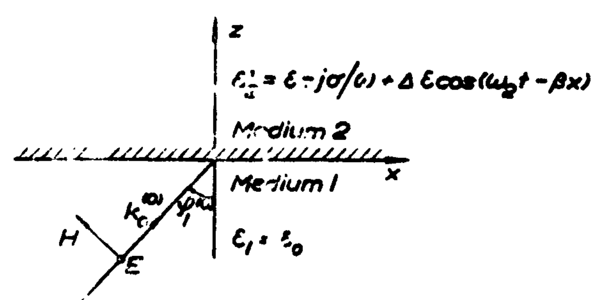


Fig. 4.1. A plane electromagnetic wave obliquely incident upon an oscillating, lossy, dielectric medium.

$$\varepsilon' = \varepsilon - j\sigma/\omega + \Delta\varepsilon\cos(\omega_2 t - \beta x) \tag{4.1}$$

where ω is the angular frequency of the wave under consideration i.e. ω_1, $\omega_1 + \omega_2$, $\omega_1 - \omega_2$, etc.

The wave equation in medium 2 is found to be, if we assume $\partial/\partial y = 0$

$$\left.\begin{aligned}&\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial z^2} - \frac{1}{c_2^2}\left(1 - j\frac{\sigma}{\omega\varepsilon}\right)\frac{\partial^2 E}{\partial t^2} = \\ &= \frac{1}{c_2^2}\frac{\Delta\varepsilon}{\varepsilon}\frac{\partial^2}{\partial t^2}\left[E\cos(\omega_2 t - \beta x)\right]\end{aligned}\right\} \tag{4.2}$$

where

$$c_2 = \sqrt{\mu_o \varepsilon}\,.$$

Using the same technique as in chapter 2 we expand the solution of equation (4.2) in a series and get with our previous notations

$$A^{(+1)} = \frac{\Delta\varepsilon}{2\varepsilon}\cdot\frac{(\omega_1 + \omega_2)^2}{c_2^2\,\beta[\beta + 2k_{x_2}^{(0)}] - \omega_2(\omega_2 + 2\omega_1) + j\,\frac{\sigma\omega_2}{\varepsilon}}\cdot A^{(0)}, \tag{4.3a}$$

$$A^{(-1)} = \frac{\Delta\varepsilon}{2\varepsilon}\cdot\frac{(\omega_1 - \omega_2)^2}{c_2^2\,\beta[\beta - 2k_{x_2}^{(0)}] - \omega_2(\omega_2 - 2\omega_1) - j\,\frac{\sigma\omega_2}{\varepsilon}}\cdot A^{(0)}. \tag{4.3b}$$

It is interesting to note that the imaginary term in the denominator will depend on the pump frequency ω_2 only and not on the frequency ω_1 of the incident wave as might be expected.

In order to obtain a solution, that satisfies the boundary conditions, we introduce the same series as in the lossless case, namely the expressions (2.4) and (2.5). The b- and R-values are then, in the first order approximation, found to be

$$\left.\begin{aligned}A^{(0)}\,b^{(0)} &= E_{inc}\,\frac{2k_o^{(0)}\cos\varphi_1^{(0)}}{k_o^{(0)}\cos\varphi_1^{(0)} + k_{z_2}^{(0)}} = \\ &= E_{inc}\,\frac{2k_{z_1}^{(0)}}{k_{z_1}^{(0)} + k_{z_2}^{(0)}},\end{aligned}\right\} \tag{4.4a}$$

$$\left.\begin{aligned}R^{(0)} &= E_{inc}\,\frac{k_o^{(0)}\cos\varphi_1^{(0)} - k_{z_2}^{(0)}}{k_o^{(0)}\cos\varphi_1^{(0)} + k_{z_2}^{(0)}} = \\ &= E_{inc}\,\frac{k_{z_1}^{(0)} - k_{z_2}^{(0)}}{k_{z_1}^{(0)} + k_{z_2}^{(0)}}.\end{aligned}\right\} \tag{4.4b}$$

$$A^{(0)}\, b^{(+1)} = E_{inc}\, \frac{\Delta\varepsilon}{2\varepsilon} \cdot \frac{(\omega_1 + \omega_2)^2}{\omega_2(\omega_2+2\omega_1) - c_2^2\beta[\beta + 2k_{x_2}^{(0)}] - j\, \frac{\sigma\omega_2}{\varepsilon}} \times$$

$$\times \frac{k_{z_1}^{(+1)} + k_{z_2}^{(0)}}{k_{z_1}^{(+1)} + k_{z_2}^{(+1)}} \cdot \frac{2k_{z_1}^{(0)}}{k_{z_1}^{(0)} + k_{z_2}^{(0)}}, \tag{4.5a}$$

$$R^{(+1)} = E_{inc}\, \frac{\Delta\varepsilon}{2\varepsilon} \cdot \frac{(\omega_1 + \omega_2)^2}{\omega_2(\omega_2+2\omega_1) - c_2^2\beta[\beta + 2k_{x_2}^{(0)}] - j\, \frac{\sigma\omega_2}{\varepsilon}} \times$$

$$\times \frac{k_{z_2}^{(0)} - k_{z_2}^{(+1)}}{k_{z_1}^{(+1)} + k_{z_2}^{(+1)}} \cdot \frac{2k_{z_1}^{(0)}}{k_{z_1}^{(0)} + k_{z_2}^{(0)}}, \tag{4.5b}$$

$$A^{(0)}\, b^{(-1)} = E_{inc}\, \frac{\Delta\varepsilon}{2\varepsilon} \cdot \frac{(\omega_1 - \omega_2)^2}{\omega_2(\omega_2-2\omega_1) - c_2^2\beta[\beta - 2k_{x_2}^{(0)}] + j\, \frac{\sigma\omega_2}{\varepsilon}} \times$$

$$\times \frac{k_{z_1}^{(-1)} + k_{z_2}^{(0)}}{k_{z_1}^{(-1)} + k_{z_2}^{(-1)}} \cdot \frac{2k_{z_1}^{(0)}}{k_{z_1}^{(0)} + k_{z_2}^{(0)}}, \tag{4.6a}$$

$$R^{(-1)} = E_{inc}\, \frac{\Delta\varepsilon}{2\varepsilon} \cdot \frac{(\omega_1 - \omega_2)^2}{\omega_2(\omega_2-2\omega_1) - c_2^2\beta[\beta - 2k_{x_2}^{(0)}] + j\, \frac{\sigma\omega_2}{\varepsilon}} \times$$

$$\times \frac{k_{z_2}^{(0)} - k_{z_2}^{(-1)}}{k_{z_1}^{(-1)} + k_{z_2}^{(-1)}} \cdot \frac{2k_{z_1}^{(0)}}{k_{z_1}^{(0)} + k_{z_2}^{(0)}}. \tag{4.6b}$$

The k- and K-values are formally the same as in the lossless case, besides the $j\sigma\omega_2/\varepsilon$ term. The k-values in medium 2, however, are in general complex quantities. The dispersion relations for the ω_1 and $\omega_1 - \omega_2$ waves in the pumped medium can now be written [compare (2.16a) and (2.16b)]

$$[k_{x_2}^{(0)}]^2 + [k_{z_2}^{(0)}]^2 = \frac{\omega_1^2}{c_2^2}\left(1 - j\,\frac{\sigma}{\omega_1\,\varepsilon}\right), \tag{4.7}$$

and

$$[k_{x_2}^{(0)} - \beta]^2 - [k_{z_2}^{(-1)}]^2 =$$

$$= \frac{(\omega_1 - \omega_2)^2}{c_2^2}\left[1 - j\,\frac{\sigma}{(\omega_1 - \omega_2)\varepsilon}\right]. \tag{4.8}$$

Since we have, in this case too, assumed medium 1 to be a vacuum, the boundary conditions require $k_{x_2}^{(0)} = k_o^{(0)}\sin\varphi_1^{(0)}$. This means that the imaginary part of k in medium 2 must be equal to the imaginary part of k_{z2}, i.e. the wave, as expected, will be attenuated in the z-direction only. Introducing

$$k_{z_2} = k_{zre_2} - jk_{zim_2}, \tag{4.9}$$

we get for the ω_1 wave

$$k_{zre_2}^{(0)} = \left\{\frac{1}{2}\left[\frac{\omega_1^2}{c_2^2} - k_o^{(0)^2}\sin^2\varphi_1^{(0)}\right] + \frac{1}{2}\sqrt{\left[\frac{\omega_1^2}{c_2^2} - k_o^{(0)^2}\sin^2\varphi_1^{(0)}\right]^2 + \frac{\omega_1^2\,\sigma^2}{c_2^4\,\varepsilon^2}}\right\}^{\frac{1}{2}}, \tag{4.10a}$$

$$k_{zim_2}^{(0)} = \left\{-\frac{1}{2}\left[\frac{\omega_1^2}{c_2^2} - k_o^{(0)^2}\sin^2\varphi_1^{(0)}\right] + \frac{1}{2}\sqrt{\left[\frac{\omega_1^2}{c_2^2} - k_o^{(0)^2}\sin^2\varphi_1^{(0)}\right]^2 + \frac{\omega_1^2\,\sigma^2}{c_2^4\,\varepsilon^2}}\right\}^{\frac{1}{2}} \tag{4.10b}$$

For the $\omega_1 - \omega_2$ wave we get similar expressions which are obtained from (4.10); ω_1 is replaced by $\omega_1 - \omega_2$ and $k_o^{(0)} \sin \varphi_1^{(0)}$ by $k_o^{(0)} \sin \varphi_1^{(0)} - \beta$.

The E-field in medium 2 now can be written

$$
\begin{aligned}
E_2 = {} & A^{(0)}b^{(0)} e^{-zk^{(0)}_{zim_2}} e^{j[\omega_1 t - xk_o^{(0)} \sin\varphi_1^{(0)} - zk^{(0)}_{zre_2}]} + \\
& + A^{(+1)}b^{(0)} e^{-zk^{(0)}_{zim_2}} e^{j\left\{(\omega_1+\omega_2)t - x[k_o^{(0)}\sin\varphi_1^{(0)} + \beta] - zk^{(0)}_{zre_2}\right\}} + \\
& + A^{(0)}b^{(+1)} e^{-zk^{(+1)}_{zim_2}} e^{j\left\{(\omega_1+\omega_2)t - x[k_o^{(0)}\sin\varphi_1^{(0)} + \beta] - zk^{(+1)}_{zre_2}\right\}} + \\
& + A^{(-1)}b^{(0)} e^{-zk^{(0)}_{zim_2}} e^{j\left\{(\omega_1-\omega_2)t - x[k_o^{(0)}\sin\varphi_1^{(0)} - \beta] - zk^{(0)}_{zre_2}\right\}} + \\
& + A^{(0)}b^{(-1)} e^{-zk^{(-1)}_{zim_2}} e^{j\left\{(\omega_1-\omega_2)t - x[k_o^{(0)}\sin\varphi_1^{(0)} - \beta] - zk^{(-1)}_{zre_2}\right\}} + \dots
\end{aligned}
\tag{4.11}
$$

Since the angles of reflection are independent of ε' these angles will be the same as in the lossless case. The angles of refraction are determined by the following expressions, using the same notations as before, for the ω_1 and $\omega_1 - \omega_2$ waves, viz.

$$
\sin \varphi_2^{(0)} = \frac{\sin \varphi_1^{(0)}}{\left\{\sin^2 \varphi_1^{(0)} + \left[\frac{k^{(0)}_{zre_2}}{k_o^{(0)}}\right]^2\right\}^{\frac{1}{2}}}, \tag{4.12}
$$

$$\left(\sin \varphi_2^{(-1)}\right)_{k_{zre_2} = k_{zre_2}^{(0)}} =$$

$$= \frac{\sin \varphi_1^{(0)} - \frac{\beta}{k_o^{(0)}}}{\left\{\left[\sin \varphi_1^{(0)} - \frac{\beta}{k_o^{(0)}}\right]^2 + \left[\frac{k_{zre_2}^{(0)}}{k_o^{(0)}}\right]^2\right\}^{\frac{1}{2}}}, \qquad (4.13a)$$

$$\left(\sin \varphi_2^{(-1)}\right)_{k_{zre_2} = k_{zre_2}^{(-1)}} =$$

$$= \frac{\sin \varphi_1^{(0)} - \frac{\beta}{k_o^{(0)}}}{\left\{\left[\sin \varphi_1^{(0)} - \frac{\beta}{k_o^{(0)}}\right]^2 + \left[\frac{k_{zre_2}^{(-1)}}{k_o^{(0)}}\right]^2\right\}^{\frac{1}{2}}} \qquad (4.13b)$$

The expressions for the transmission and reflection coefficients will formally be the same as in the lossless case, but the R-, A-, and b-values given in (4.3)-(4.6) must, of course, be used.

Finally we want to study the values of conductivity and frequencies for which the losses must be taken into account. For the simpler case $\beta = 0$, and $\varepsilon = \varepsilon_o$ we can write $R^{(-1)}$ like

$$\frac{\Delta\varepsilon}{2\varepsilon_o}\,\frac{R^{(-1)}}{E_{inc}} = \frac{\left(1-\frac{\omega_2}{\omega_1}\right)^2}{\frac{\omega_2}{\omega_1}\left(\frac{\omega_2}{\omega_1}-2\right)+j\,\frac{\omega_2}{\omega_1}\cdot\frac{\sigma}{\omega_1\varepsilon_o}} \cdot \frac{2\cos\varphi_1^{(0)}}{\cos\varphi_1^{(0)}+\sqrt{\cos^2\varphi_1^{(0)}-j\,\frac{\sigma}{\omega_1\varepsilon_o}}}\,\times$$

$$\times\,\frac{\sqrt{\cos^2\varphi_1^{(0)}-j\,\frac{\sigma}{\omega_1\varepsilon_o}}\;\mp\;\sqrt{\left(1-\frac{\omega_2}{\omega_1}\right)^2-\sin^2\varphi_1^{(0)}+j\left(1-\frac{\omega_2}{\omega_1}\right)\frac{\sigma}{\omega_1\varepsilon_o}}}{\pm\sqrt{\left(1-\frac{\omega_2}{\omega_1}\right)^2-\sin^2\varphi_1^{(0)}}\;\pm\;\sqrt{\left(1-\frac{\omega_2}{\omega_1}\right)^2-\sin^2\varphi_1^{(0)}+j\left(1-\frac{\omega_2}{\omega_1}\right)\frac{\sigma}{\omega_1\varepsilon_o}}} \tag{4.14}$$

where the upper sign is valid when $\omega_1 > \omega_2$ and the lower when $\omega_2 > \omega_1$. In Fig. 4.2 are shown some curves of $R^{(-1)}$ for different values of σ/ω_1 (cf Fig. 3.4)

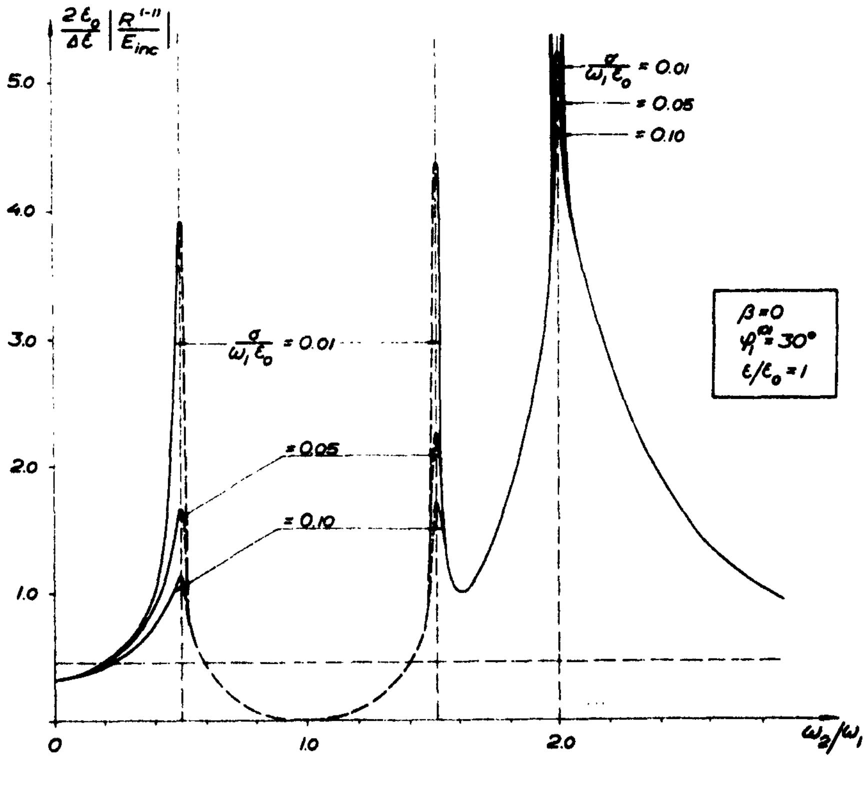


Fig. 4.2. The first order amplitude $R^{(-1)}$ of the reflected $\omega_1 - \omega_2$ wave as a function of ω_2/ω_1 when $\beta = 0$, and $\varphi_1^{(0)} = 30^o$, assuming different values of σ/ω_1.

With the exception of the frequency range at or near the resonance points, where the σ-term is the remaining part of the denominator in (4.14), the influence of the losses is small if $\sigma/\omega_1 \varepsilon_o \ll 1$. For a value of σ of 10^{-4} mhos/meter, $\sigma/\omega_1 \varepsilon_o$ will be less than 0.01 for frequencies above 200 Mc/s. On the other hand, if σ is not greater than $5 \cdot 10^{-13}$ mhos/meter the same term will be < 0.01 for frequencies > 1 c/s.

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